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## Analysing the vibration in simply supported beam

Mayank Mangal

Department of Mechanical Engineering  
SRM University Chennai (India)

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### General Note

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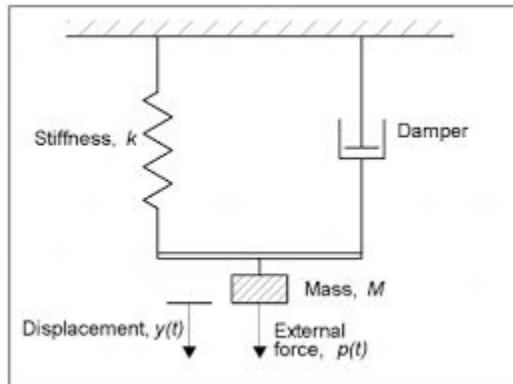
### ABSTRACT

A vast amount of published work can be found in the field of beam vibrations dealing with analytical and numerical techniques. This paper deals with the vibration in a simply supported beam (SSB) having varying mass. This problem is generally faced by engineers at the time of construction of bridges, vibration in axle shaft of rail road car in order to withstand the forced vibration due to moving objects and other factors. This paper involves the implementation of Meirovitch equation <sup>[1]</sup> of motion to obtain the frequency of simply supported beam having mass varying in 'x' direction. This approach involves the application of boundary condition <sup>[3]</sup> to get generalised result for the frequency of SSB.

**Keywords:** vibrations in (SSB), Varying mass, Meirovitch equation, boundary condition, frequency of SSB

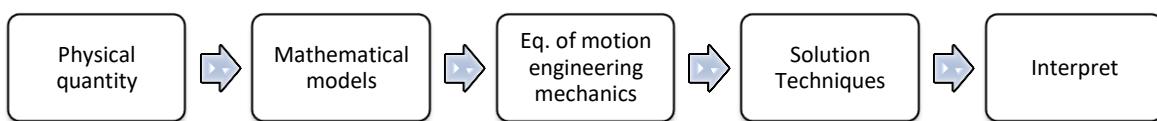
## 1. INTRODUCTION

It is well known that beams are very common type of structural elements which can be classified according to their geometric configuration as cantilever, simply supported beam, overhanging double overhanging. Since Analyzing the vibration of beam is one of the important issues in structural engineering application mostly in long-span bridges. Practically, simply supported beams transfer more load to the end supports than a continuous beam, which is advantageous in certain situations. And also simply supported beams may have to be larger / deeper to carry the same load, which increases their shear capacity. Design of such structures to resist dynamic forces, such as wind and earthquakes, requires knowledge of their natural frequencies. The vibration of simply supported beam in vertical plane finds wide application.



**Figure 1**

Basic equipments of vibration



**Figure 2**

Flow diag. For analysis of vibration

For simply supported beam of varying mass  $m[x]$  analysis, Meirovitch equation <sup>[1]</sup> of motion is used. The vibration analysis that has been done here is a process of describing a structure in terms of its natural frequency. We are familiar with term Vibration <sup>[2]</sup> is a

mechanical phenomenon is also known as oscillation. Which is defined as the back and forth movement of a body about a point known as equilibrium point. These oscillations can be periodic or random it directly depends on the case.

Now these vibrations cause sound effects or heat with the body and finally cause mechanical deformation. But the result which we get through this paper can solve the above problem.

Basic equipments required to measure vibration are mass, spring and damper as shown in fig1.

In order to measure vibration, the process shown above must be followed.

### Examples of vibration

- Musical instruments
- Vibration of mobile phone
- Vocal cords ...

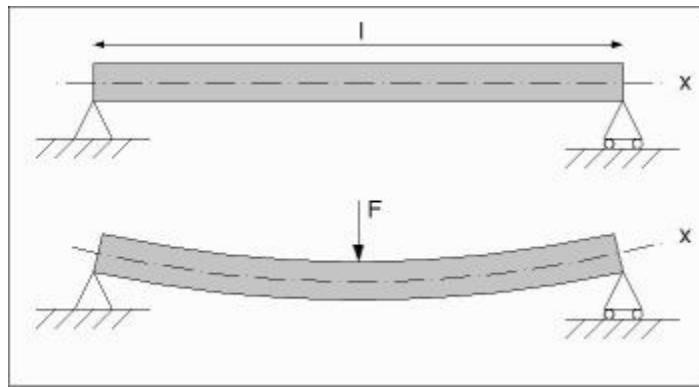
### Types of vibration

Free vibration- When a mechanical body starts to vibrate with an initial input without any application of force. Example small angular displacement on pulling a chid back.

Forced vibration- When the vibration is a function of time which can be harmonic or non harmonic. Example shacking of washing machine.

## 2. MATHEMATICAL ANALYSIS

Let us consider simply supported beam of length 'L' as shown below is subjected to a varying mass  $[m(x)]$ .



**Figure 3**

Application of load on SSB

Here,  $F=m(x)$

According to Meirovitch equation<sup>[1]</sup> of motion of a vibrating mass is given by,

$$D^2y/Dx^2 \{ EI(D^2y/Dx^2) \} = w^2 \cdot m(x) \cdot y(x)$$

Where E is the modulus of rigidity of the material, I is the moment of inertia,  $y(x)$  is the displacement in vertical direction and  $w$  is angular velocity through we can find the frequency of the vibrating simply supported beam.

Let us apply the boundary condition<sup>[3]</sup> on the given setup.

At,

$$x=0, y(x)=0, \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=0 \quad \text{----- (2)}$$

Similarly at

$$x=L, y(x)=0, \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=0 \quad \text{----- (3)}$$

And at

$$x=L/2, y(x)=A, \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=0$$

For uniform and free vibration equation (1) can be written as,

$$\frac{d^4y}{dx^4} = [w^2/EI] \cdot m(x) \cdot y(x)$$

Now, let us suppose  $\beta^4 = [w^2/EI] \cdot m(x)$ 

The mode shape of a simply supported beam is,

$$Y(x) = A \{ (\sin \beta_n L - \sinh \beta_n L) \cdot (\sin \beta_n x - \sinh \beta_n x) + (\cos \beta_n L - \cosh \beta_n L) \cdot (\cos \beta_n x - \cosh \beta_n x) \}$$

Where,  $n=1, 2, 3, \dots$ 

$$\& \beta L = n\pi$$

$$w^2 = (\beta^4 \cdot EI) / m(x)$$

$$w = (n \cdot \pi / L)^2 \cdot [EI / m(x)]^{1/2}$$

And, for natural frequency we can use.

$$F = w / (2\pi)$$

### 3. SOLVED EXAMPLE

Illustration-Consider a simply supported beam of length 12.5m has modulus of elasticity be  $2.1 \times 10^{11}$  (N/m<sup>2</sup>) and moment of inertia be  $4.5 \times 10^{-11}$  m<sup>4</sup> and mass distribution m(x) along length is

$$m(x) = 3x^2 + \log x \text{ Find the frequency at a distance 5 m from left end.}$$

Solution

Given,

$$L = 12.5 \text{ m}$$

$$E = 2.1 \times 10^{11} \text{ (N/m}^2\text{)}$$

$$I = 4.5 \times 10^{-11} \text{ m}^4$$

$$\& m(x) = 3x^2 + \log x$$

$$\text{The value of mass at } x=5 \text{ is } m(x) = 3(5)^2 + \log 5$$

$$m(5) = 75.69 \text{ kg}$$

$$w = (n \cdot \pi / L)^2 \cdot [EI / m(x)]^{1/2}$$

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$$w = (1540.5625) \cdot (0.353) \quad \text{for } n=1$$

$$w = 544.3 \text{ rad/sec}$$

We know,

$$f = w / (2\pi)$$

Then,

$$f = (544.3) / (2\pi)$$

$$f = 86.6 \text{ Hz}$$

#### 4. CONCLUSION

The natural frequency of varying mass of a SSB is computed in this paper numerically. This technique will be attractive for beams having varying masses under the application of load. The boundary condition for SSB are studied and applied in Meirovitch equation. And the result obtained is defined by a sample problem as done above.

#### FUTURE WORK

Future work is to define the frequency of vibration of beam having point load which is varying with respect to a point.

#### REFERENCES

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